Simulation of impact failure of rock by numerical manifold method

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Abstract: Impact failure of rock is a kind of typical dynamic problem. In order to perfectly analyze the dynamic problem by use of numerical manifold method (NMM), the Newmark method of dynamic finite element method was adopted to improve the algorithm in original NMM on the basis of analyzing the solution idea of dynamics problem in NMM. There were three obvious advantages in the improved method over the original one: ①The solution of this method was convergent without any condition when suitable parameters were selected; ②The time step adopted in this method was much longer than that in the original one; ③The damping effect in the dynamic problem was fully considered. In the end, a calculation example was used to illustrate the application of this method in simulation of the whole process of impact failure of rock, which overcame the shortcomings of FEM in simulating the block movement after rock failure.

Key words: rock; impact failure; numerical manifold method (NMM); block movement; dynamics

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岩石冲击破坏的数值流形方法模拟

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摘 要：岩石冲击破坏是一种典型的动力学问题。为了更好地利用数值流形方法对动力学问题进行分析，本文在对原数值流形方法中的动力学问题求解思想进行分析的基础上，采用动力有限元方法中的 Newmark 法对该算法进行了改进。改进后的数值流形方法与原来相比具有 3 个明显的优势：①当选择合适的参数后，该方法能够保证解的无条件收敛；②可以采用比原算法大的多的时间步长；③充分地考虑了动力学问题中的阻尼效应。最后通过一个算例说明了改进后的数值流形方法能够很好地模拟岩石在冲击载荷作用下破坏的全过程，克服了有限元法不能模拟岩石破坏后块体运动情况的不足。

关键词：岩石；冲击破坏；数值流形方法；块体运动；动力学

0 Introduction

Numerical manifold method (NMM) is a newly numerical simulation method proposed by Doctor Shi Gen-hua after the method of DDA [1-2]. Although it has only been there for a little more than ten years after it was put forward in the early of 1990's, it has attracted much attention from the beginning because of its advantages. Up to now, it has been widely used in analysis of rock and soil engineering. However, although NMM has certain application in rock and soil engineering and preliminarily shows its advantages, its application scope is still relatively less than that of FEM and DEM in solution of practical problems because of short period of time after its birth. But because NMM adopts two sets of meshes—mathematical mesh and physical mesh, it has obvious advantages in dealing with occurrence and propagation of crack, and it has been widely used in this field [3].

Meanwhile, this method is a kind of numerical analysis method absorbing the advantages of FEM and DDA, especially in simulation of block movement after failure of materials. It fully absorbs the theory of block movement in DDA, and can perfectly simulate block movement process after failure. It is an important breakthrough comparing with FEM based on continuous

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media, which overcomes the shortcomings that FEM can only give the stress distribution in the study domain and cannot simulate its fragmentation and block movement after failure and so on. But now the relevant references simulating block movement process after rock impact failure are not available. The block movement process after failure belongs to the obvious dynamics problem. Therefore, on the basis of discussing the calculation method of dynamics problem in NMM, this paper improves this method and uses the improved program to simulate the rock impact failure process.

1 Dynamics solution format in NMM

1.1 Dynamics solution idea in original NMM

The failure problem of rock under impact load belongs to the obvious fracture dynamics problem. Fracture dynamics studies the fracture mechanics problem in which the inertia effect cannot be ignored [4]. The solution method of fracture dynamics problem is obviously different from that of fracture static problem, between which the most difference is the inertia effect being not ignored. Fracture mechanics considering the inertia effect is fracture dynamic mechanics or dynamic fracture mechanics. When the dynamic load is acted, the body not only produces elastic-plastic deformation, but also the particles in the body will obtain certain acceleration, and have inertia force, which is so called inertia effect under dynamic load [5].

In analysis of dynamics problem, NMM introduces inertia matrix that is correspondent with the mass matrix in FEM to fully consider the inertia effect in dynamics problem. In analyzing the dynamics problem, NMM adopts the different method from the dynamic FEM. In brief, the most difference between dynamic and static problems in NMM is that during the calculation of current time step every element inherits the velocity of previous time step, while in dealing with the static problem, the velocity of every element of the current time step is set as zero. This treatment method is too easy for the average dynamics problem, and it only considers the element's inertia effect and does not consider its damp, which is not very accurate to the average dynamic problem[6]. Therefore, on the basis of analyzing the dynamics solution format in NMM, this paper makes some improvements in order to simulate the average dynamics problem better.

In the solution formula of NMM, the primary difference between dynamics and statics problem lies in mass matrix, which is a very important matrix in dynamics problem. When the time step is short, the inertia matrix will control the movement and stability of every particle in the material. In the original NMM, the solution method of the dynamics problem is introduced in detail in relative references[1-2].

From these references, it can be seen that this kind of dynamics algorithm is explicit. It obtains displacement and velocity of next time step by making use of those of current time step, that is to say, in calculating dynamics problems by NMM, it inherits the displacement and velocity of previous time step at the beginning of current time step.

From above, it can be seen that it is an explicit algorithm. That is to say in NMM, the dynamics computation inherits the velocity of previous time step. While in this method, the displacement is unfolded with Taylor series whose truncation error is third order. When the time step is longer, its error becomes larger. While the algorithm is convergent with condition, the selection of the time step must be shorter than a certain value[7]: $\Delta t \le \Delta t_{\mathrm{cr}} = \dfrac{T_n}{\pi}$, where: T_n is the inherent vibration cycle of the smallest element in the solution system, therefore, the dimension of the smallest element will determinate the selection of time step. This algorithm is more suitable to solve the wave propagation problem, because on one hand the solution method is completely correspondent with the property of wave propagation, on the other hand shorter time step is needed in study on the wave propagation problem which completely meets with the requirements of this algorithm. However, it is not much suitable to the average structure dynamics problem because the low frequency in the structure dynamic response is most, and longer time step is allowed to use. So in view of the solution idea of dynamic FEM, the Newmark method is adopted to improve the solution method of the average structure dynamics problem in NMM.

1.2 NMM adopting Newmark method

In FEM, the solution equation to dynamics problem is structure dynamics equation [8-9] :

$$M\ddot{d} + C\dot{d} + Kd = F \quad , \tag{1}$$

where, M is the mass matrix; C is the damp matrix; d is the displacement increment; $\dot{d}$, $\ddot{d}$ are the velocity and acceleration, respectively; $K = K_{\rm e} + K_{\rm cn} + K_{\rm cs} + K_{\rm f}$, $K_{\rm e}$ is the stiffness matrix, $K_{\rm cn}$, $K_{\rm cs}$ are the contact matrixes of block and discontinuous face respectively, $K_{\rm f}$ is the constraint matrix; F is the total load vector, $F = F_{\rm p} + F_{\rm b} + F_{\rm f} - F_0 + F_{\rm cn} + F_{\rm cs} + F_{\rm fr}$, where $F_{\rm p}$ is the extern load vector, $F_{\rm b}$ is the body force vector, $F_{\rm f}$ is the equivalent load vector caused by the constraint displacement, F_0 is the initial stress vector, $F_{\rm cn}$, $F_{\rm cs}$ are the equivalent load vector caused by the normal and shear contact respectively, and $F_{\rm fr}$ is the equivalent load vector caused by friction between contact faces.

To the average structure dynamics problem, dynamics equation (1) is usually solved by Newmark method. When suitable parameters are selected, Newmark method is a stable numerical method without any condition, and longer time step can be adopted, which is more suitable to solve the structure dynamics problem. Therefore, in this paper, equation (1) is solved by Newmark method. In fact, Newmark integration method is generalized from linear acceleration method, and it has the following hypothesis:

$$d_{t+\Delta t} = d_t + \dot{d}_t \Delta t + \left[(\frac{1}{2} - \alpha)\ddot{d}_t + \alpha \ddot{d}_{t+\Delta t} \right] \Delta t^2$$

$$\dot{d}_{t+\Delta t} = \dot{d}_t + \left[(1-\beta)\ddot{d}_t + \beta \ddot{d}_{t+\Delta t} \right] \Delta t \quad , \tag{2}$$

where, α, β are two parameters relevant to integration accuracy and stability. When $\beta \geq 0.5$, $\alpha \geq 0.25(0.5+\beta)^2$, Newmark method is a stable method without any condition. In this method, the displacement of time $t+\Delta t$ is obtained from the equation $M\ddot{d}_{t+\Delta t} + C\dot{d}_{t+\Delta t} + Kd_{t+\Delta t} = F_{t+\Delta t}$.

When suitable parameters are selected, Newmark method is a stable method without any condition, that is to say the time step Δt does not influence the stability of solution, and is only relevant to the accuracy of the solution. By contrast with the original solution method in NMM, this method obtains much longer time step at the cost of solving the inverse of $\hat{K}$. Meanwhile, this method fully considers damp effect in dynamics problem.

1.3 Failure criterion and calculation of stress intensity factor

The response process of the material under impact load is one in which the new crack occurs, develops and the blocks form. Therefore, the criterion of the new crack's occurrence and the existing crack's propagation is very important. Because rock is a typical brittle material, the Mohr-Coulomb constitutive model is adopted in NMM.

For the initiation of new cracks, a stress-based criterion should be considered. The Mohr-Coulomb's law with three parameters is taken as the failure criterion for new cracks. It is assumed that new cracks start to appear if: (a) the first principle stress is larger than the tensile strength of the material, or (b) the maximum shear stress is larger than the shear strength of the material. Take σ_1 and σ_3 to indicate the first and third principal stresses, the failure criterion can then be expressed as[12-13]: ① tensile failure: $\sigma_1 = T_0$. ② shearing failure:

$$\begin{cases} \dfrac{\sigma_1 - \sigma_3}{2} = c, \text{if } \dfrac{\sigma_1 + \sigma_3}{2} > 0 \quad (0 < \sigma_1 < T_0) \ , \\ \dfrac{\sigma_1 - \sigma_3}{2} = c\cos\varphi - \dfrac{\sigma_1 + \sigma_3}{2} \sin\varphi \ , \\ \text{if } \dfrac{\sigma_1 + \sigma_3}{2} < 0 \quad (0 < \sigma_1 < T_0) \ , \end{cases} \tag{3}$$

where T_0 is the tension strength of the material, c is the cohesion and φ is the friction angle.

For existing cracks, fracture mechanics regards that whether the crack propagates or not is decided by not the stress value of the crack tip but the stress intensity factor. So the calculation of the stress intensity factor is very important. Some references adopt the second manifold method and the sub-region boundary element method to calculate the stress intensity factor [12-13]. Suppose the fracture toughness of the material is $K_{\rm IC}$, the maximum circumferential stress theory is adopted to determine the direction of crack growth measured from the current crack line θ:

$$K_{\rm I} \sin\theta_0 + K_{\rm II}(3\cos\theta_0 - 1) = 0 \quad , \tag{4}$$

The fracture criterion for mix mode problem takes the form as

$$\cos\frac{\theta_0}{2}\left(K_{\rm I}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{\rm II}\sin\theta_0 \right) = K_{\rm IC} \quad 。 \tag{5}$$

2 A calculation example and its analysis

2.1 Mechanical characteristics of blasting shock failure and load simplification

The shock load has many kinds. In order to connect with the actual problem on one hand and make comparison with the actual results on the other hand, the blasting load is chosen as the shock load, and the corresponding actual problem is the formation process of blasting funnel in a finite domain with a circle charge blasting in the two-dimensional space. The rock will cause the following mechanical response after charge detonates [10]: ①The high shock wave pressure will cause the rock around the blasthole crushed and produce compressive crush and initial cracks; ②The circular tension stress and reflected tension stress will cause crack propagation and further fragmentation of rock including formation of initial cracks and second propagation of cracks; ③The blasting gas makes rock cut into pieces by cracks and move to form blasting funnel.

When the dynamite explodes, the energy released by the dynamite acts on the rock in the form of high pressure shock waves, and comes into being the stress waves propagating outside in the rock. Furthermore, it will cause the movement of the rock particle, and produce the corresponding velocity and acceleration, which is a typical dynamic process. Meanwhile, if the intensity of the stress wave exceeds the dynamic tensile strength or shear strength of the rock, the new crack will occur in the rock. Under extern force, the cracks will further propagate, run through, and finally cut the rock into blocks. So this process can be perfectly simulated by the improved dynamics NMM introduced in the paper.

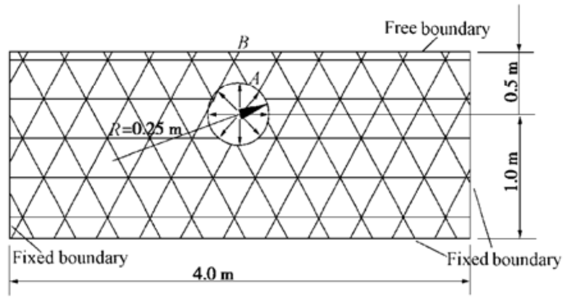
In fact, the rock blasting process is very complicated, therefore, on the premise of meeting with engineering requirement, the rock blasting process can be dealt with in a simple way. The blasting load is simplified as a shock triangle wave acted on inner wall of the blasthole, whose maximum pressure is 5 GPa. The duration of pressure rising is 80 ms, and that of unloading is 220 ms.

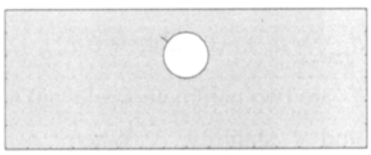
2.2 Calculation model and analysis of calculated results

The calculation model established in this paper is the formation process of blasting funnel when circular charge in the two-dimensional finite domain rockmass explodes. Supposing the dimension of the calculation model is: length×width=4 m×1.5 m, the diameter of the circle is 0.5 m, and the coordinate of the blasthole center is (0,0). *A* and *B* are the two measured points lying up the blasthole, whose coordinates are (0, 0.25) and (0, 0.5) respectively. The scheme of the calculation model is shown as Fig. 1(a).

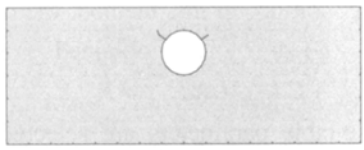
In this calculation example, the type of analyzed problem is dynamic. Therefore, the calculation parameters of material should also be dynamic. The rock's dynamic elastic modulus, dynamic ratio, density and dynamic fracture toughness, friction angle, cohesion strength and dynamic tension strength are respectively 80 GPa, 0.20, 2400 kg/m^3, 0.5 MN/m$^{3/2}$, 45°, 10 MPa and 30 MPa.

According to the above parameters, the failure process of rock under shock load is simulated. Eight instantaneous conditions in the calculation process are shown as Fig.1 (b)~(g).

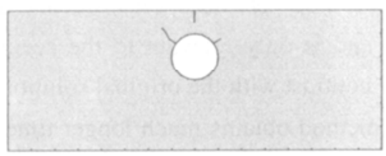




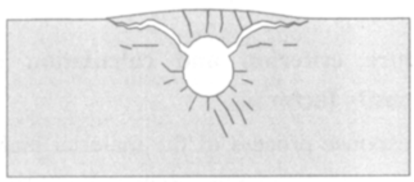
(b) t = 0.3 ms



(c) t = 0.4 ms



(d) t = 0.5 ms



(e) t = 3.0 ms

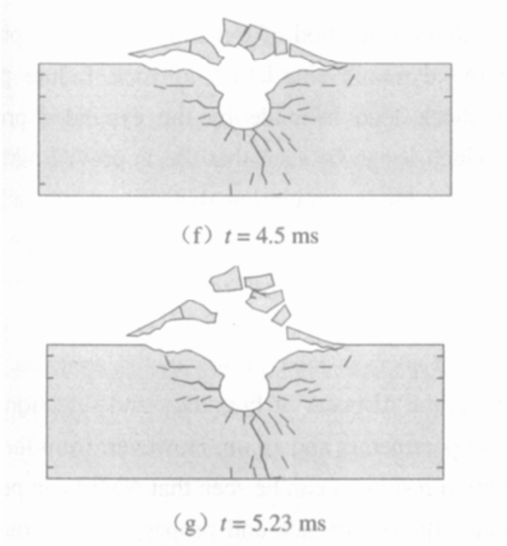


Fig.1 Calculation model and calculated results

In order to perfectly analyze the blasting process, the horizontal displacement *U* and vertical displacement *V* of two points *A* and *B* are recorded and plotted in Fig. 2. At the same time, the calculation of the stress intensity factor (SIF) is vital in simulation of fracture. From the simulation results of Fig.1, it can be seen that the two main cracks are dominative in controlling the formulation of the blasting funnel. The variation law of the SIF of the two cracks with time are shown in Fig. 3.

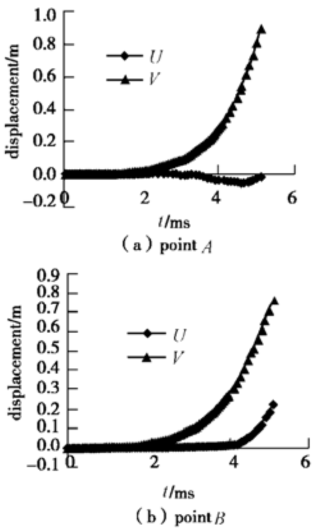


Fig. 2 Displacements of the two points

From the simulation results, it can be seen that:

(1) Failure form of materials. Under shock load, crack first occurs and propagates from the wall of the blasthole. It can be obviously seen that crack first occurs along the direction of 45° from vertical. According to the mechanical theory of materials, shear stress is largest in this direction, and the material is prone to be sheared to failure. While in this calculation, compressive failure is ignored, and the failure type of materials is classified as shear failure and tension failure. Obviously, the failure form of materials belongs to shear failure in the beginning.

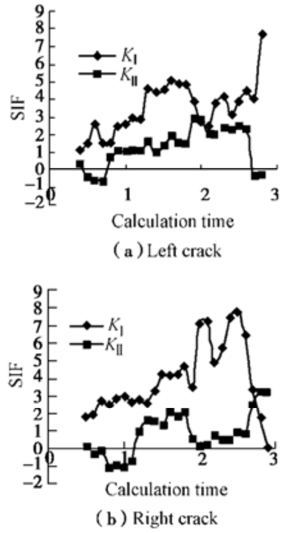


Fig. 3 Variation of SIF of the right and left cracks with time
（calculation time: ms; SIF: $10^7 N.m^{-3/2}$）

(2) Occurrence location of cracks. Crack firstly occurs along the direction of 45° from vertical, and then appears above the blasthole vertical with the free face. It shows that rock near free face is pulled to failure by tension stress caused by reflection of stress waves.

(3) Influence of free face to the development of cracks. Free face is vital in affecting the development of crack. Towards free face, development predominance is very obvious, which is because the existence of free face supplies the space for rock fragmentation. Crack easily propagates in this direction while in other three directions propagation of crack is very slow, even basically not.

(4) Failure form of this model. The shape of failure is very much like that of blasting funnel, an invert triangle. It shows that the simulation result is very correspondent with the actual condition on one hand, on the other hand the failure form of rock mainly belongs to shear failure, and development predominance of two main cracks is very obvious.

(5) Fragmentation condition of rock. Fragmentation region mainly lies in the region above the blasthole, where resistance is least and rock is very easy to be fallen into pieces. While in other regions, the failure degree of rock is rather less and no large crack occurs, but only some small cracks exist and get no development.

(6) Development of crack. Bifurcation phenomenon

exists in crack development, that is to say, crack does not develop along the initial path, but deviates from the original path and changes into two or more branch cracks, or even new small cracks occur where the original crack does not reach. It is the most common phenomenon in the dynamic propagation of crack[11]. In rock blasting, this bifurcation phenomenon of crack should be utilized to make rock fully fallen into pieces.

(7) Destruction of the boundary. At the three fixed boundaries, a certain number of cracks occur. From simulation results, it can be seen that these cracks do not cause by the propagation of the cracks from the blasthole, but caused by the reflection of stress wave from free face. In theory, these three fixed boundaries should be changed into non-reflected ones in order to observe the formulation of the blasting funnel. But now the disposal method of boundary is not as perfect as FEM, and the above simulation happens. It is an important aspect to be improved in future.

(8) Movement of block after fragmentation. In dealing with this problem, NMM completely inherits the advantages of DDA in simulating movement and contact of blocks, which makes no penetration and tension in all contact positions during the block movement.

(9) From the displacement curves of the two measure points A and B in Fig.2, horizontal displacement and vertical displacement are produced at these two points under the action of the shock load inside because these two points are above the blasthole. However, the vertical displacement is far larger than that of the horizontal, which shows that the rock above the blasthole mainly flies up.

(10) From Fig. 3, it can be seen that the propagation of the two main cracks both belongs to mixed crack propagation of type Ⅰand Ⅱ. At the same time, the first stress intensity factor K_{I} is larger than second stress intensity factor K_{II}, therefore the crack propagation of type Ⅰ is dominant. The variation of the stress intensity factor with time is not monotonous, which indicates that the crack does not belong to unstable propagation, and it needs external force to drive it during the propagation.

3 Conclusions

On the basis of discussion of the solution idea of dynamic problem in original NMM, this paper makes use of this solution method of dynamic FEM to improve it. Then the dynamic simulation of rock failure process under shock load is made by the expanded program, from which it can be seen that the improved NMM has much potential in simulating dynamic shock failure of materials. But it should be seen that because the period after the birth of NMM is not long, there are some practical problems to be studied in specific practical application such as rock failure in the area near to the blasthole, the disposal of boundary and selection of the relevant parameters and so on. However, from the above simulation results, it can be seen that NMM can perfectly simulates the occurrence and propagation of crack, and rock fragmentation and block movement after the action of shock load, which may supply a new thought for forecasting of blasting fragment. On the basis of other researchers' work, this paper introduces the theory and algorithm of NMM into simulation of rock under shock failure. Although there are some questions to be solved, development foreground should be affirmative and it may be helpful to initiate a new path to the simulation of rock failure under shock load.

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