Active earth pressure on circular shaft lining obtained by simplified slip line solution with general tangential stress coefficient
竖井衬砌主动土压力在一般环向压应力系数下的简化滑移线解

Y.M. Cheng (郑榕明) [1], HU Ya-yuan (胡亚元) [2]

(1.Department of Civil and Structural Engineering, Hong Kong Polytechnic University, Hong Kong, China;2.Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China)

Abstract: In this paper, a general tangential stress coefficient is introduced to overcome the limitations of the Haar & Von Karman hypothesis in axi-symmetric earth pressure problem. A simplified analytical solution of active earth pressure on circular shaft with no wall friction and horizontal backfill is developed in the present paper. It is demonstrated that the tangential stress coefficient has a major effect on the active pressure and the Harr & Von Karman hypothesis may be unacceptable in practice. The authors consider that an active earth pressure based on a tangential stress coefficient equal to the coefficient of earth pressure at rest is suitable for engineering practice.

Key words: axi-symmetric; tangential stress coefficient; active pressure, slip line

CLC number: TU 432 **Document code:**A **Article ID:**1000－4548(2005)01－0110－06

Biography: Cheng Yung-ming(1973－), male, Ph D. Associate Professor, Hong Kong Polytechnic University.

摘 要：引进一个广义环向压应力系数来修正空间轴对称滑移线问题中的哈尔-卡门(Haar-Von Karman)完全塑性假定，采用简化滑移线方法，在竖井衬砌后背光滑，水平填土的条件下，得到竖井衬砌上主动土压力的解析式。分析表明：环向压应力系数对竖井衬砌上的主动土压力有较大的影响，若采用哈尔-卡门完全塑性假定，其所求得的主动土压力最小，在工程应用中偏于危险。笔者建议在工程应用中采用环向压应力系数为静止土压力系数 K_0 时所求得的竖井衬砌上的主动土压力值。

关键词：轴对称；环向压应力系数；主动土压力；滑移线

0 Introduction

For circular excavation, most of the engineers in Hong Kong and in other countries adopt plane strain active pressure coefficient in the analysis and design, but this approach is obviously not correct and conservative. For circular excavation, the stress states are axi-symmetric in nature and the active pressure may attain a maximum value with increasing depth which is greatly different from thet results in plane strain case. For circular foundation and excavation, the Haar-Von Karman's hypothesis is adopted by many researchers to simplify the analysis. Berezantzev[1] adopted simplified slip line to study the active pressure solution. Steinfeld[2] and Karafiath[3] have assumed an axisymmetric Coulomb-type failure surface where the sliding mass is a cone and the total earth pressure is obtained in a way similar to the classical Coulomb's method. Lorenz[4] adapted Steinfeld's theory to the active pressure, while neglecting the tangential stress in radial direction equilibrium. All of the above works are based on the use of Harr and Von Karman's hypothesis where λ=1. Prater[5] summarized the above works and adopted a tangential stress coefficient equal to K_0 and K_a which is different from the Harr-Von Karman's hypothesis in investigating the active pressure for shaft lining with Coulomb's method. Prater viewed that Berezantzev's solution delivered very low value of active earth pressure because of Haar-Von Karman's hypothesis. Cui[6] also considered that the active earth pressure from Berezantzev's solution may be risky to be applied in practice. The tangential stress coefficient initiated by Prater is extended to modify Haar-Von Karman hypothesis in present paper. Using the simplified assumptions similar to that of Berezantzev[1], analytical formulae are also developed in the present study. From the results of analysis, many interesting results for axisymmetric problems which differ greatly with the plane strain solution are obtained in the present paper.

1 Method of characteristics for axisym -metric Problem

The equilibrium equations for a toroidal element (Fig.1) can be written in cylindrical coordinates r, θ, z

Recevied date:2003－11－21

as:

$$\frac{\partial\sigma_{\rm r}}{\partial r}+\frac{\partial\tau_{\rm rz}}{\partial z}+\frac{\sigma_{\rm r}-\sigma_\theta}{r}=0\quad, \tag{1a}$$

$$\frac{\partial\tau_{\rm rz}}{\partial r}+\frac{\partial\sigma_z}{\partial z}+\frac{\tau_{\rm rz}}{r}=\gamma\quad, \tag{1b}$$

The four stress components can be expressed in terms of the mean stress σ and the inclination angle ψ that is formed by extending the major principal stress to the r axis (Fig. 2b) as:

$$\sigma_{\rm r}=\sigma(1+\sin\varphi\cos 2\psi)-c\cdot\cot\varphi\quad, \tag{2a}$$

$$\sigma_z=\sigma(1-\sin\varphi\cos 2\psi)-c\cdot\cot\varphi\quad, \tag{2b}$$

$$\tau_{\rm rz}=\sigma\sin\varphi\sin 2\psi\quad, \tag{2c}$$

$$\sigma_\theta=\lambda\sigma_1=\lambda\sigma(1+\sin\varphi)-\lambda c\cdot\cot\varphi\quad, \tag{2d}$$

and $\sigma=(\sigma_1+\sigma_3)/2+c\cdot\cot\varphi$, c is cohesive strength and the tangential coefficient λ is a ratio between σ_θ and σ_1. λ is taken as 1.0 by most of the researchers in the past for simplicity which is known as the Harr and Von Karman hypothesis but this ratio should lie somewhere between 1 and K_a and should be taken as a variable in general. Parter[5] has considered $\lambda=K_0=1-\sin\varphi$ and K_a in his analysis with limit equilibrium method.

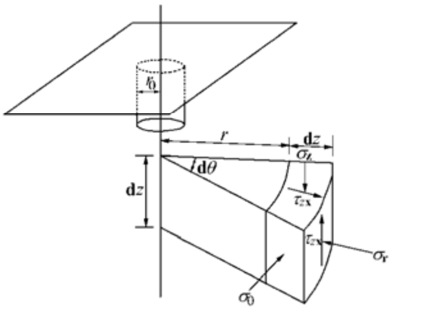

Fig. 1 Cylindrical coordinate system and stress components

The characteristic lines for the solution are the α and β lines on which the shear strength is fully mobilized. In literature they are often referred to as slip lines, but their significance are related to the equations of equilibrium instead of displacement. The geometry dictates the slopes of the slip lines that can be written as:

$$\frac{{\rm d}z}{{\rm d}r}=\tan(\psi+m\mu)\ \ \text{where}\ \ \mu=\frac{\pi}{4}-\frac{\varphi}{2}\quad, \tag{3}$$

and m takes the value –1 for an α line and +1 for an β line (shown in Fig.2(b)). These equilibrium and yield equations form a set of hyperbolic partial differential equations which will reduce to two ordinary differential equations expressing the changes in stress along each characteristic line in terms of the changing inclination ψ and position (r, z)

$${\rm d}\sigma+m2\sigma\tan\varphi{\rm d}\psi+\frac{(1-\lambda-\lambda\sin\phi)\sigma-c(1-\lambda)\cot\phi}{r}{\rm d}r+ m\frac{\lambda(1+\sin\varphi)\sigma+c(1-\lambda)\cot\phi}{r}\tan\varphi{\rm d}z=\gamma(m\tan\varphi{\rm d}r+{\rm d}z)\ . \tag{4}$$

To simplify the study, the variables will be normalized with the circular shaft radius in the form:

$$\Omega=\frac{\sigma}{r_0\gamma},\ \ R=\frac{r}{r_0},\ \ Z=\frac{z}{r_0},C=\frac{c}{r_0\gamma},Q_0=\frac{q_0}{r_0\gamma}\ , \tag{5}$$

where r_0 is the radius of the circular shaft, z is the vertical depth under consideration, γ is the unit weight of soil, q_0 is the external surcharge. Substituting eqn.(5) into eqn.s(3) and (4), we have:

$$\frac{{\rm d}Z}{{\rm d}R}=\tan(\psi+m\mu)\ \ \text{where}\ \ \mu=\frac{\pi}{4}-\frac{\varphi}{2}\quad, \tag{6}$$

$${\rm d}\Omega+m2\Omega\tan\varphi{\rm d}\psi+\frac{(1-\lambda-\lambda\sin\phi)\Omega-C(1-\lambda)\cot\varphi}{R}{\rm d}R+ m\frac{\lambda(1+\sin\varphi)\Omega+C(1-\lambda)\cot\phi}{R}\varphi\tan\varphi{\rm d}Z=m\tan\varphi{\rm d}R+{\rm d}Z. \tag{7}$$

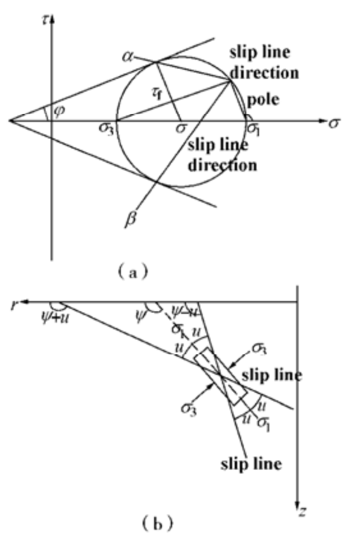

Fig. 2 (a) Mohr circle under failure condition. (b) Sign convention and notation

2 Simplified slip line solution

If we assume the slip lines to be straight lines in the R-Z plane which are the assumptions as used by Berezantzev, the inclination of β slip line is:

$$m=1,\ \ \psi=\frac{\pi}{2}={\rm const}\ \ \text{and}\ \ {\rm d}Z=\tan(\frac{3\pi}{4}-\frac{\varphi}{2}){\rm d}R\ , \tag{8}$$

Put eqn.(8) into eqn.(4), we obtain

$$\frac{{\rm d}\Omega}{{\rm d}R}-\left[\lambda\frac{(1+\sin\varphi)^2}{\cos^2\varphi}-1\right]\frac{\Omega}{R}-\frac{C(1-\lambda)}{R}\frac{1+\sin\varphi}{\sin\varphi\cos\varphi}=-\frac{1}{\cos\varphi}\ , \tag{9}$$

Let

$$\eta=\lambda\frac{(1+\sin\varphi)^2}{\cos^2\varphi}-1=\lambda\tan^2(\frac{\pi}{4}+\frac{\varphi}{2})-1\quad, \tag{10a}$$

$$\xi=\frac{1-\lambda}{\eta}\tan^2(\frac{\pi}{4}+\frac{\varphi}{2})+1\quad. \tag{10b}$$

Put eqn.(10) into eqn.(9), and the solution of differential equation is:

$$\Omega = aR^{\eta} + \frac{R}{(\eta-1)\cos\varphi} - \frac{C(1-\lambda)(1+\sin\varphi)}{\eta\sin\varphi\cos\varphi} \quad . (11)$$

Eqns.(2a)~(2d) are normalized with the shaft radius r_0 and reduce to the following forms with eqn. (8):

$$\Omega_R = \Omega(1-\sin\varphi) - C\cdot\cot\varphi \quad , \quad (12a)$$

$$\Omega_Z = \Omega(1+\sin\varphi) - C\cdot\cot\varphi \quad , \quad (12b)$$

$$\Omega_\theta = \lambda\Omega(1+\sin\varphi) - \lambda C\cdot\cot\varphi \quad , \quad (12c)$$

According to eqn.s(12a)~(12c)and(11), the component of stress tensor can be express as following:

$$\Omega_R = a(1-\sin\varphi)R^{\eta} + \frac{(1-\sin\varphi)}{(\eta-1)\cos\varphi}R - \cot\varphi\frac{(1+\eta-\lambda)}{\eta}C \quad ,(13a)$$

$$\Omega_Z = a(1+\sin\varphi)R^{\eta} + \frac{(1+\sin\varphi)}{(\eta-1)\cos\varphi}R - \xi C\cdot\cot\varphi \quad ,(13b)$$

$$\Omega_\theta = ak(1+\sin\kappa)R^{\eta} + \frac{(1+\sin\varphi)}{(\eta-1)\cos\varphi}kR - \xi kC\cdot\cot\varphi \quad ,(13c)$$

Here R_b denote the intersect of the β slip line which pass through the point (1, Z) with R axis, so

$$R_b = 1 + Z\cdot\tan(\frac{\pi}{4}-\frac{\varphi}{2}) \quad . \quad (14)$$

On ground surface, $\Omega_z|_{R=R_b} = Q_0$, the integral constant is then determined as:

$$a = \frac{Q_0 + \xi C\cot\phi}{R_b^{\eta}(1+\sin\phi)} - \frac{1}{(\eta-1)\cos\phi R_b^{\eta-1}} \quad . (15)$$

Put eqn.(15) into eqn.s(13a)~(13c) and notice that:

$$\frac{1-\sin\varphi}{1+\sin\varphi} = \tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) \ , \quad \frac{1-\sin\varphi}{\cos\varphi} = \tan(\frac{\pi}{4}-\frac{\varphi}{2}) \ .$$

Eqn.s(13a)~(13c) will be simplified to:

$$\Omega_R = R\frac{\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{\eta-1}[1-(\frac{R}{R_b})^{\eta-1}] + Q_0(\frac{R}{R_b})^{\eta}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) + C[\xi(\frac{R}{R_b})^{\eta}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) - \frac{1-\lambda+\eta}{\eta}]\cot\varphi \quad , \quad (16a)$$

$$\Omega_Z = R\frac{\tan(\frac{\pi}{4}+\frac{\varphi}{2})}{\eta-1}[1-(\frac{R}{R_b})^{\eta-1}] + Q_0(\frac{R}{R_b})^{\eta} + C\xi[(\frac{R}{R_b})^{\eta}-1]\cot\varphi \quad , \quad (16b)$$

$$\Omega_\theta = \lambda R\frac{\tan(\frac{\pi}{4}+\frac{\varphi}{2})}{\eta-1}[1-(\frac{R}{R_b})^{\eta-1}] + \lambda Q_0(\frac{R}{R_b})^{\eta} + \lambda\xi C[(\frac{R}{R_b})^{\eta}-1]\cot\varphi \quad , \quad (16c)$$

In eqn.(16.1), put R=1, the active earth pressure on shaft lining is:

$$P_a = \frac{\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{\eta-1}(1-\frac{1}{R_b^{\eta-1}}) + Q_0\frac{1}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) - C[\frac{1-\lambda+\eta}{\eta} - \frac{\xi}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2})]\cot\varphi \quad . \quad (17)$$

So the actual active pressure can be obtained by eqn.(5) as:

$$p_a = \gamma r_0\frac{\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{\eta-1}(1-\frac{1}{R_b^{\eta-1}}) + q_0\frac{1}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) - c[\frac{1-\lambda+\eta}{\eta} - \frac{\xi}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2})]\cot\varphi \quad , \quad (18)$$

where

$$R_b = 1 + Z\cdot\tan(\frac{\pi}{4}-\frac{\varphi}{2}) = \frac{r_0 + z\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{r_0} .$$

When $\lambda = 1$, eqn.(10) becomes:

$$\eta = \tan^2(\frac{\pi}{4}+\frac{\varphi}{2}) - 1 = 2\tan\varphi\tan(\frac{\pi}{4}+\frac{\varphi}{2}) , \quad \xi = 1 , \ (19)$$

and eqn.(18) becomes:

$$p_a = \gamma r_0\frac{\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{\eta-1}(1-\frac{1}{R_b^{\eta-1}}) + q_0\frac{1}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) - c[1-\frac{1}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2})]\cot\varphi \quad . \quad (20)$$

Eqn.(20)is the same as Berezentzav's original formula for axi-symmetric active pressure. Eqn.s(17) and (18) are more general in that λ is now a variable. According to eqn.(18), the active earth pressure can be formulated as:

$$p_a = K_{a\gamma}\gamma z + K_{aq}q_0 - K_{ac}c \quad , \quad (21)$$

where the active earth pressure coefficients $K_{a\gamma}$, K_{aq}, K_{ac} on shaft lining are defined as:

$$K_{a\gamma} = \frac{\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{\eta-1}(\frac{r_0}{z} - \frac{r_0}{zR_b^{\eta-1}}) = \frac{\tan(\frac{\pi}{4}-\frac{\varphi}{2})}{\eta-1}\{\frac{1}{Z} - \frac{1}{Z[1+Z\tan(\frac{\pi}{4}-\frac{\varphi}{2})]^{\eta-1}}\} , \quad (22a)$$

$$K_{aq} = \frac{1}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2}) = \tan^2(\frac{\pi}{4}-\frac{\varphi}{2})\frac{1}{[1+Z\tan(\frac{\pi}{4}-\frac{\varphi}{2})]^{\eta}} \quad , \quad (22b)$$

$$K_{ac} = [\frac{1-\lambda+\eta}{\eta} - \frac{\xi}{R_b^{\eta}}\tan^2(\frac{\pi}{4}-\frac{\varphi}{2})]\cot\varphi = \{\frac{1-\lambda+\eta}{\eta} - \frac{\xi\tan^2(\frac{\pi}{4}-\frac{\varphi}{2})}{[1+Z\tan(\frac{\pi}{4}-\frac{\varphi}{2})]^{\eta}}\}\cot\varphi \quad . \quad (22c)$$

Because of arching action effect, the axisymmetric active earth pressure must be smaller than or equal to plane strain active pressure. For the first term of eqn.(18), the quantity η must satisfy:

$$\eta - 1 \ge -1 \quad . \tag{23}$$

Put eqn. (23) in eqn.(10), we obtain:

$$\lambda \ge \tan^2(45 - \frac{\varphi}{2}) = K_a \quad . \tag{24}$$

Hence the tangential stress ratio λ will lie between 1.0 and K_a. Using eqn.s (17) and (18), the upper and lower estimates of the active pressure can be determined by putting λ =1.0 and K_a.

Obviously, when r_0 tends to infinity, an axi-symmetric problem will degenerate to a plane strain problem and Z tends to zero. According to eqn.s (22a)~(22c), we can obtain coefficients of active earth pressure in plane strain problem as:

$$K_{a\gamma} = K_{aq} = \tan^2(\frac{\pi}{4} - \frac{\varphi}{2}), \quad K_{ac} = 2\tan(\frac{\pi}{4} - \frac{\varphi}{2}).$$

3 Special considerations in axi-symmetric active pressure

Eqn.s(22a)~(22c) are not applicable when $\eta = 0$ or $\eta = 1$ or $\varphi = 0$ which has been neglected by Berezantzev in the past. Considering firstly the case of $\eta = 0$, according to eqn.(10), we get

$$\lambda = K_a = \tan^2(\frac{\pi}{4} - \frac{\varphi}{2}) \quad . \tag{25}$$

Take eqn.s(22a)~(22c) to limit $\eta \to 0$ and replace it by eqn.(25), we get

$$K_{a\gamma} = K_{aq} = \tan^2(\frac{\pi}{4} - \frac{\varphi}{2}) \quad , \tag{26a}$$

$$K_{ac} = 2\tan(\frac{\pi}{4} - \frac{\varphi}{2})\{1 + \ln[1 + Z\tan(\frac{\pi}{4} - \frac{\varphi}{2})]\} . \tag{26b}$$

Secondly, for the case of $\varphi = 0$, according to eqn.(24) and considering that λ is less than or equal to 1.0, we get λ=1 and $\eta = 0$.

Put $\varphi = 0$ into eqn.(26a)~(26b), we obtain :

$$K_{a\gamma} = K_{aq} = 1, \quad K_{ac} = 2[1 + \ln(1 + Z)] \quad . \tag{27}$$

Thirdly we consider the case of $\eta = 1$. According to eqn.(10), we get

$$\varphi = 2\arctan\sqrt{\frac{2}{\lambda}} - \frac{\pi}{2} = \varphi_{cr} \quad . \tag{28}$$

If η tends to 1.0, put eqn.(29) into eqn.s (22a)~(22c), we get

$$K_{a\gamma} = \sqrt{\frac{\lambda}{2}}\frac{r_0}{z}\ln\frac{r_0 + z\sqrt{\lambda/2}}{r_0} = \sqrt{\frac{\lambda}{2}}\frac{\ln(1 + Z\sqrt{\lambda/2})}{Z}, \tag{29a}$$

$$K_{aq} = \frac{\lambda}{2}\frac{r_0}{r_0 + z\sqrt{\lambda/2}} = \frac{\lambda}{2}\frac{1}{1 + Z\sqrt{\lambda/2}}, \tag{29b}$$

$$K_{ac} = \frac{\sqrt{2\lambda}(r_0 + 2z\sqrt{\lambda/2})}{r_0 + z\sqrt{\lambda/2}} = \frac{\sqrt{2\lambda}(1 + 2Z\sqrt{\lambda/2})}{1 + Z\sqrt{\lambda/2}} . \tag{29c}$$

According to eqn.(21), we obtain

$$p_a = \gamma r_0\sqrt{\frac{2}{\lambda}}\ln\frac{r_0 + z\sqrt{\lambda/2}}{r_0} + q_0\frac{\lambda}{2}\frac{r_0}{r_0 + z\sqrt{\lambda/2}} - c\frac{\sqrt{2\lambda}(r_0 + 2z\sqrt{\lambda/2})}{r_0 + z\sqrt{\lambda/2}} \quad . \tag{30}$$

According to the first term of eqn.s(18) and (30), it is concluded that if $\varphi > \varphi_{cr}$ (see eqn.(28)), active earth pressure induced by self weight of soil attains a limiting value as z tends to infinity. If $\varphi \le \varphi_{cr}$, the active pressure will tend to infinity as z tends to infinity. When λ=1, the critical angle φ_{cr} is approximately 19.5° while the general critical angle φ_{cr} will increase with decreasing λ in accordance with eqn.(28).

Finally for the case of $\lambda = K_0 = 1 - \sin\varphi$ as adopted by Prater using Coulomb's method, from eqn.(10) we can write

$$\eta = \sin\varphi, \quad \xi = \sec^2(\frac{\pi}{4} + \frac{\varphi}{2}) \quad . \tag{31}$$

Put eqn.(31) to eqn.s(22a)~(22c), we obtain

$$K_{a\gamma} = \frac{\tan(\frac{\pi}{4} - \frac{\varphi}{2})}{(1 - \sin\varphi)Z}\{[1 + Z\tan(\frac{\pi}{4} - \frac{\varphi}{2})]^{1 - \sin\varphi} - 1\}, \tag{32a}$$

$$K_{aq} = \frac{1}{[1 + Z\tan(\frac{\pi}{4} - \frac{\varphi}{2})]^{\sin\phi}\varphi}\tan^2(\frac{\pi}{4} - \frac{\varphi}{2}), \tag{32b}$$

$$K_{ac} = \{2 - \frac{1}{[1 + Z\tan(\frac{\pi}{4} - \frac{\varphi}{2})]^{\sin\varphi}}\sec^2(\frac{\pi}{4} - \frac{\varphi}{2})\}\cot\varphi . \tag{32c}$$

According to the eqn.(21), we obtain as

$$p_a = \gamma r_0\frac{\tan(\frac{\pi}{4} - \frac{\varphi}{2})}{1 - \sin\varphi}(R_b^{1 - \sin\varphi} - 1) + q_0\frac{\tan^2(\frac{\pi}{4} - \frac{\varphi}{2})}{R_b^{\sin\varphi}} - c[2 - \frac{1}{R_b^{\sin\varphi}}\sec^2(\frac{\pi}{4} - \frac{\varphi}{2})]\cot\varphi \quad . \tag{33}$$

According to the first term of eqn.(33), the active earth pressure induced by the self weight of soil will tend to infinity if Z tends to infinity. Prater considered that the active pressure based on tangential stress coefficient K_0 is acceptable in engineering use[5].

4 Characteristics of active earth pressure on shaft lining

After normalization by the radius of shaft, $K_{a\gamma}$, K_{aq}, K_{ac} are formulated in terms of the dimensionless depth Z but is independent on the shaft radius. $K_{a\gamma}$, K_{aq}, K_{ac} are hence expressed in terms of dimensionless depth Z but not the shaft radius. Computations of $K_{a\gamma}$, K_{aq}, K_{ac} for different tangential stress coefficient λ are shown in Fig. 3 – 5. It can be seen that the influences of λ on $K_{a\gamma}$, K_{aq}, K_{ac} are very significant and cannot be simply taken as 1.0 in Harr-Von Karman's hypothesis. The smaller is λ, the greater will be the values of $K_{a\gamma}$, K_{aq}, K_{ac}. If λ is greater than K_a, $K_{a\gamma}$, K_{aq} will tends to zero while Kac will tends to a maximum value as the dimensionless depth Z tends to infinity. If λ is equal to K_a, according to eqn.(26), $K_{a\gamma}$, K_{aq} are constant and are equal to K_a while K_{ac} will tend to infinity as dimensionless depth Z tends to infinity.

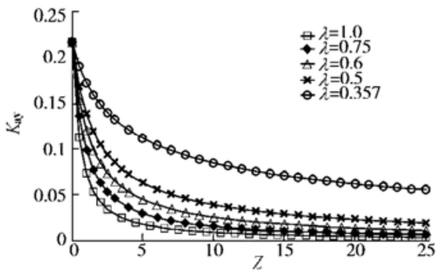

Fig. 3 Variation of $K_{a\gamma}$ with dimensionless depth Z and λ for $\varphi=40^{\circ}$

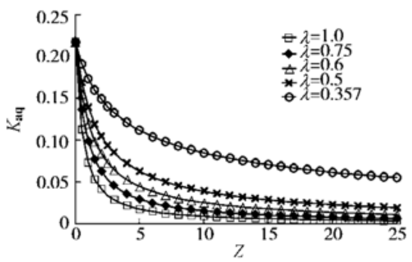

Fig. 4 Variation of K_{aq} with dimensionless depth Z and λ for $\varphi=40^{\circ}$

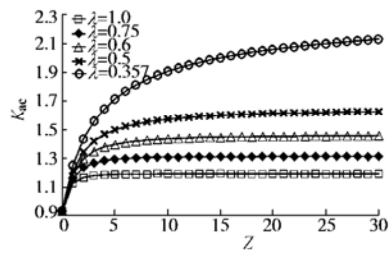

Fig. 5 Variation of K_{ac} with dimensionless depth Z and λ for $\varphi=40^{\circ}$

Fig. 6 – 8 show the variation of $K_{a\gamma}$, K_{aq}, K_{ac} with dimensionless depth Z for various internal friction angle while λ is set to at-rest earth pressure coefficient ($K_0 = 1-\sin\varphi$). The smaller φ is, the greater will be the values of $K_{a\gamma}$, K_{aq}, K_{ac} which is obvious. If φ is greater than zero, $K_{a\gamma}$, K_{aq} will tend to zero and K_{ac} will tend to a maximum value as the dimensionless depth Z tends to infinity. If φ is equal to zero, $K_{a\gamma}$, K_{aq} are constant and equal to unity while K_{ac} tends to infinity as the dimensionless depth Z tend to infinity.

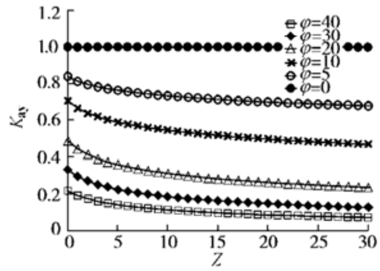

Fig. 6 Variation of $K_{a\gamma}$ with dimensionless depth Z and φ for λ=K_0

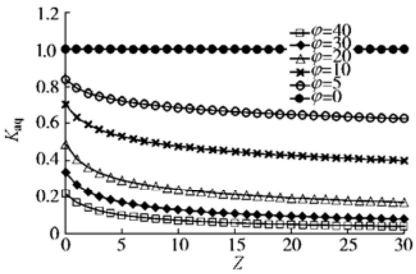

Fig. 7 Variation of K_{aq} with dimensionless depth Z and φ for λ=K_0

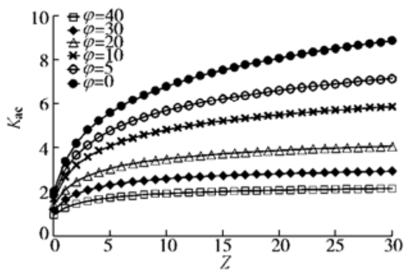

Fig. 8 Variation of K_{ac} with dimensionless depth Z and φ for λ=K_0

In actual practice, the active earth pressure induced by self weight of soil is the most important consideration and the critical internal friction angle φ_{cr} with the tangential stress coefficient λ is shown in Fig. 9 which is defined by eqn.(28). If the internal friction angle is smaller than or equal to that critical friction angle φ_{cr} with a specific λ, the earth pressure induced by the self weight of soil will tend to infinity as dimensionless depth Z increases. Beyond that, the active pressure due to self weight of soil will however attain a maximum value as Z tends to infinity. The variation of earth pressure with dimensionless Z for some λ at a critical friction angle φ_{cr} is shown in Fig.10.

5 Conclusions

Field measurements have shown that the original

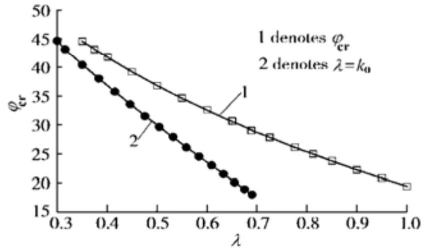

Fig 9 Variation of φcr and K_0 with tangential stress coefficient λ

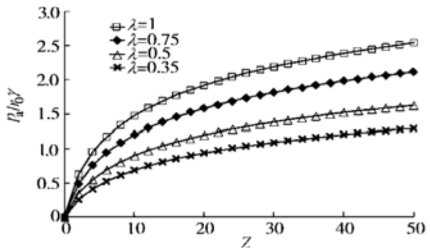

Fig. 10 Variation of $p_a / r_0\gamma$ with dimensionless depth Z for different φ_c K_0 in figure

equation by Berezantzev is not correct which may be a reason that Berezantzev's solution is seldom adopted in practice. The tangential stress coefficient λ initiated by Prater is introduced in the present study and Berezantzev's solution is modified in the present paper. The active earth pressure is formulated and the results show that Berezantzev's original theory where λ=1delivers the lowest value of earth pressure which may be risky in shaft lining design. The modified solutions in the present paper can be adopted to overcome the weakness of Berezantzev's formulation.

The results in Fig. 3–5 for active earth pressure coefficient $K_{a\gamma}$, K_{aq}, K_{ac} have shown that these coefficients are very sensitive to the assumed tangential stress coefficient λ acting on radial planes. If λ is greater than K_a, $K_{a\gamma}$, K_{aq} will tend to 0 while Kac will attain a maximum value with increasing Z. If $K_{a\gamma}$, K_{aq} are both equal to K_a which is earth pressure coefficient for plane strain problem, $K_{a\gamma}$, K_{aq} are constant and are equal to K_a while K_{ac} will tend to infinity as dimensionless depth Z tends to infinity. Eqn.(24) shows that λ must be greater than the active Rankine value of plane strain for realistic axi-symmetric results. However, if λ is set equal to unity as is originally assumed by Berezantzev's theory, simplified slip line method will produce the minimum active earth pressure which is not safe. To be on the safe side, Prater suggested that λ should be set equal to the earth pressure coefficient at rest K_0. The present formulation allows any tangential stress coefficient to be adopted. More importantly, the range of active pressure can be estimated.

Fig. 6–8 shows that the internal friction angle φ takes an important role on active earth pressure coefficient $K_{a\gamma}$, K_{aq}, K_{ac}. It is found that the greater is the internal friction angle φ, the greater will be the arch action. If φ is set equal to 0, $K_{a\gamma}$, K_{aq} are both equal to 1.0 which is the same as the active earth pressure coefficient for plane strain problem. The earth pressure coefficients should generally be calculated with eqn.(18) and eqn.s(22a)–(22c) while the corresponding formulae for particular cases are also provided in the present paper.

Attention was paid to the active earth pressure induced by the self weight of soil. Fig. 9 shows that the critical angle φ_{cr} is closely related to the tangential stress coefficient λ. If the realistic internal friction angle is smaller than critical angle, the earth pressure caused by self weight does not attain a maximum limit as Z increases. If λ is set equal to the earth pressure coefficient at rest $K_0(=1-\sin\varphi)$, as is shown in first term of eqn.(34), the earth pressure induced by itself weight will not be limited if Z tends to infinity which more differ from Berezantzev's results. The authors consider that active earth pressure based on a tangential stress coefficient K_0 (the coefficient of earth pressure at rest) is suitable for engineering.

References:

[1] Berezantzev V G. Earth pressure on the cylindrical retaining wall[J]. Proc Brussels Conf on Earth Pressure Problems, 1958, (2):21-27.

[2] Steinfeld D K. Über den Erddruck auf Schacht-und Brunnenwandungen[A]. Contribution to the foundation engineering meeting[C]. Hamburg. German Soc Of Soil Mech Found Eng 1958, 111-26.

[3] Karafiath L. On some problems of earth pressure[A]. Acta Tech Acad Hung[C]. Budapest, 1953, 327-337.

[4] Lorenz H. Offene Senkkästen. Grundbautaschenbuch. W. Ernst & Son, 1966, 795-798.

[5] Prater E G. An examination of some theories of earth pressure on shaft lining[J]. Canadian Geotechnical Journal, 1977, **14**(1): 91-106.

[6] Cui Guang-xin. Loading of shaft lining for deep alluviam[J]. Chinese Journal of Geotechnical Engineering, 2003, **25**(3): 294-296.