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Limit analysis of soft ground reinforced by geosynthetics
土工合成物加强软土地基的极限分析

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Abstract The stability of soft ground reinforced by geosynthetics is currently analyzed by conventional circular arc method. In this paper it is confirmed that the circular arc slide is not a kinematically admissible failure mode, and the only possible way of failure of the reinforced ground is lateral spreading of soil under the geosynthetic layer. Accordingly, a simple wedge equilibrium method has been proposed for the design of such ground.
Key words geosynthetics, soft ground, limit analysis.
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文 摘 对土工合成物加强软土地基,目前仍用圆弧滑动法分析其稳定性。本文根据极限分析的原理,判定圆弧滑动不是一种可能的破坏方式.只有侧向挤出才符合真实的破坏机理.并在此基础上提出一种基于滑块平衡的简便分析方法。
关键词 土工合成物,软土地基,极限分析。
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1 Introduction

It is a popularly used measure to strengthen the soft ground under embankment dams by one or two layers of geosynthetics (geotextile or geogrid) put on the ground surface. This method is not only easy to be realized and does not damage the initial soil structure, but also is effective on the increase of stability and reduction of settlement, therefore it promises good prospect of future development. Until now the only way available in the design of such reinforced ground is using the conventional circular arc method, but according to this method many successfully constructed projects have a factor of safety less than 1[1]. The comparison of stability calculation between non-reinforced and reinforced ground also demonstrates little effect of reinforcement, and only 0.02– 0.03 is predicted in the increase of the factor of safety due to reinforcement, which is obviously inconsistent with the experiences gained from practice. Therefore the appropriateness of the circular arc method for this kind of ground is doubtful.

In the circular arc analysis there are three approaches in the calculation of resisting moment induced by the geosynthetics as follows (Fig. 1)

$$M_T = RT\cos\theta \tag{1a}$$

$$M_T = RT(\cos\theta + \sin\theta \cdot \mathrm{tg}\,\varphi) \tag{1b}$$

$$M_T = RT \tag{1c}$$

As indicated by Liu et al.[2], the main drawback of the traditional method remains in its inability to model the shear stress at the soil-geotextile interface, but the proposal offered by them to take account of the shear stress can not be considered as a successful one. A universally accepted way to deal with this problem is using FEM, but it seems too troublesome for practical use. Still the third way recently become popular is regarding the ground as linear or non-linear Winkler's foundation, in which only vertical settlement is considered[3]. Analysis by this method shows that if the geosynthetics has undergone a large deformation there will be some shear stress induced. But in our opinion, this approach is far away from reality, because it disregards the lateral movement of soil which just is the major cause of soft foundation failure.

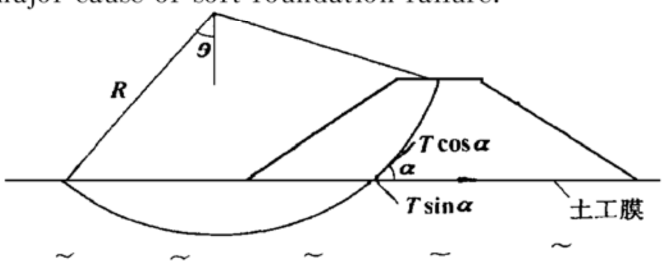

Fig. 1 Circular arc method

The aim of this work is to seek an alternative of the circular arc method which is simple enough and can be readily accepted by engineers.

2 Mechanism of foundation failure

According to the theory of limit equilibrium a strict solution of failure problem must be both statically and kinematically admissible. A solution is statically admissible when only the equilibrium equations, non-yield requirement and stress boundary conditions are satisfied. A solution is kinematically admissible when only the flow law of plasticity and velocity boundary condition are satisfied, or expressed in more simple way, when the mechanism of failure is considered. A fairy amount of examples show that when a slide surface quite resemble to the true solution is adopted, the corresponding kinematically admissible solution will be very close to the true solution. Therefore, to gain a correct solution of failure problem, the examination of failure mechanism is of primary concern.

First of all let us consider the change of possible velocity field before and after the reinforcement. The circular arc method postulates a rotational failure covering

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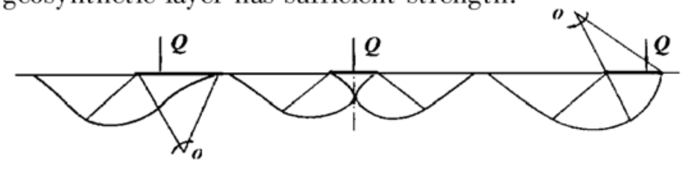
both the foundation soil and embankment soil which is only kinematically admissible before the reinforcement. But after the reinforcement, if the geosynthetics keeps its integrity, the embankment body can not be broken in two parts. Because the limit equilibrium theory makes a rigid plastic body assumption, the deformation before failure is not considered. Therefore, the only possible mode of failure after reinforcement is the movement of embankment body together with the geosynthetic layer as a whole. For the non-smooth interface between the geosynthetics and foundation soil, there are 3 possible modes of failure: forward tilting, subsidence and backward tilting, as shown in Fig. 2. The embankment dams usually have nearly symmetric cross-section, therefore, except for special case when external horizontal load must be taken into account, the only possible mode of failure is the subsidence of embankment accompanying with lateral spreading of foundation soil if the geosynthetic layer has sufficient strength.



(a) Forward tilting (b) Subsidence (c) Backward tilting

Fig. 2 Possible failure modes

3 Solution by method of characteristic line

The limit equilibrium equation can be solved mathematically by the method of characteristic line. For the problem of rigid foundation failure the solution was obtained by Shen (1962)[4] as follows (Fig. 3).

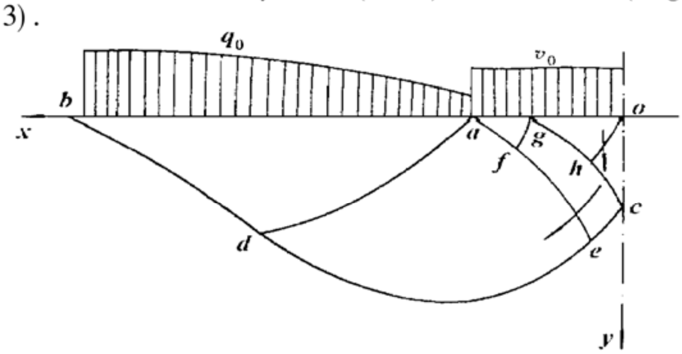


Fig. 3 Analysis of rough foundation

The static analysis is carried out first in the following sequence: ① starting from the boundary ab where the surface load q_0 is prescribed, the stress field in $\triangle abc$ can be obtained by solving Cauchy problem; ②solve Goursat (or Riemann) problem to obtain the stress field in fan area ade; ③determine the stress in $\triangle afg$ by solving mixed problem based on the friction condition at interface ag; ④again solving Goursat problem gives the stress in $\square cefg$; ⑤determine the stress in $\triangle och$ by solving mixed problem based on the condition of symmetry at the center line; ⑥the solution of Goursat problem finally determines the stress in $\triangle ogh$ and the normal and shear stress at surface og. The whole interface oa must be divided into two section: ag, where the friction has been totally mobilized and og, where partial mobilization of friction is observed. The mid point g is determined by the condition of symmetry at point c.

The kinematical analysis is carried out in reverse sequence: ① obtain velocity field in $\triangle ogh$ by solving Cauchy problem based on the prescribed velocity on og; ② solve mixed problem based on the zero velocity in horizontal direction at oc to obtain velocity field in $\triangle och$; ③determine velocity on the characteristic line ec and solving Goursat problem in $\square cefg$; ④solve mixed problem in $\triangle afg$ based on the prescribed vertical velocity on ag; ⑤determine velocity on line ed and solve Goursat problem in fan area ade; ⑥ finally determine velocity on bd and solve Goursat problem in $\triangle adb$.

An example of practical calculation is shown in Fig. 4, where φ_0 is the friction angle at interface, and $\bar{q}$ and $\bar{\tau}$ are normalized normal and tangential stress, the actual limit load is calculated according to following equation

$$q = (c + q_0 \mathrm{tg}\,\varphi)\,\bar{q} - c \cdot \mathrm{ctg}\,\varphi \tag{2}$$

The corresponding half width of foundation is $\bar{B} = 3$. The actual value is $B = (c + q_0 \mathrm{tg}\,\varphi)\,\bar{B}/\gamma$, γ is the unit weight of soil.

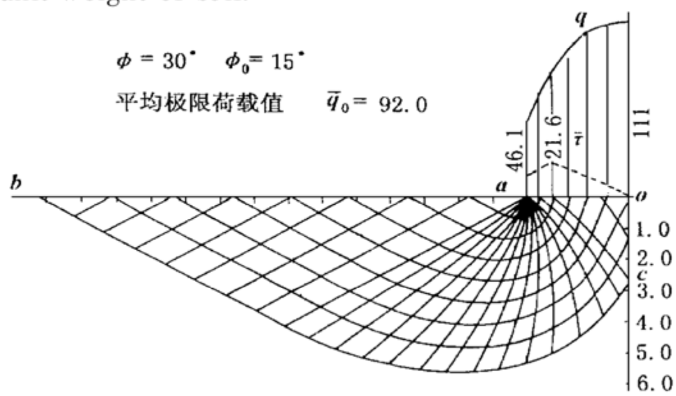


Fig. 4 Limit load on rough foundation

4 Effective consolidation stress method

The theory of limit equilibrium deals with the stress and strain rate in the limit state. But for saturated soft soil it is barely possible to predict the pore pressure under the limit state. Therefore it seems inadequate to use the effective stress concept for predicting failure of such soil. A reasonable approach is to estimate the pore pressure in the consolidation state under design load and then to carry out the calculation based on the assumption that the shear strength of soil remains unchanged from the consolidation state to the limit state[5]. In this way the shear strength of clay is calculated as follows

$$s = s_0 + \Delta\sigma_c' \,\mathrm{tg}\,\varphi_c \tag{3}$$

where φ_c is determined by direct shear test, s_0 is the shear strength of natural soil, and $\Delta\sigma_c'$ is the increase of effective consolidation stress at rupture surface. When φ_{cu} obtained from the consolidated undrained triaxial test is available

$$\mathrm{tg}\,\varphi_c = \frac{\sin\varphi_{cu}}{1 - \sin\varphi_{cu}} \tag{4}$$

φ_c is usually larger than φ_{cu} by 2°− 3°. A simple version is to use the vertical consolidation stress $\Delta\sigma_z'$ instead of $\Delta\sigma_c'$

$$s = s_0 + \Delta\sigma_z' \cdot \mathrm{tg}\,\varphi_{cu} \qquad (3a)$$

According to this approach both σ_c' and σ_z' vary in the space, s will be also a variable. Therefore the saturated clay can be regarded as a material with a variable cohesion. A computer program has been developed to solve the equation of limit equilibrium of such soil by characteristic line method[6]. Fig. 5 shows a computed example of rough foundation. Its total width is 10m and the foundation soil is assumed to be completely consolidated under design load 50kPa. The mean value of limit load is 102.4kPa with a maximum shear stress 13.88kPa.

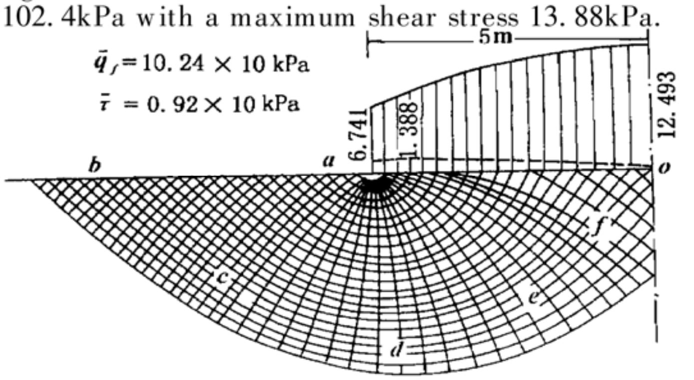


Fig. 5 Rough foundation on soft ground

5 Kinematically admissible solution

The method of characteristic line is quite complicated for practical use. The kinematically admissible solution is a much simpler way to predict the limit load. There are some different approaches to get kinematically admissible solution, such as wedge equilibrium method, kinematical element method and energy method[7]. The wedge equilibrium method includes force equilibrium and momentum equilibrium. The classical earth pressure theory and circular arc method are just the examples of this method. In the following the force equilibrium method will be developed to solve the problem.

Let us consider an ideal soil with constant shear strength s. Using method of characteristic line, Prandtl gave the theoretical value of limit load $q = (2+\pi)s$ (Fig. 6(a)). If one wedge or two wedges or three wedges is used instead of fan zone acd, then according to the force equilibrium the kinematically admissible solution will be 6.000s, 5.313 s and 5.150s respectively, as shown in Fig. 6 (b), Fig7(a) and Fig. 6(c). The three-wedge solution yields an error as low as 0.2%. Meanwhile the circular arc method gives a solution of $q = 5.52s$ as shown in Fig. 6(d). The simple example demonstrates that if an adequate slide surface is chosen, the accuracy of kinematically admissible solution is quite sufficient.

Now turn our attention to the problem of reinforced ground. Referring to the actual sliding body bounded by surface $bcdefo$ in Fig. 5, a sliding system with 5 wedges will be postulated as shown in Fig. 8. The total friction force is assumed to be $T = 0.5T_f$, where T_f is limit value of shear resistance when the friction along the interface has been fully mobilized. This assumption is equivalent to the hypothesis that $\tau = s$ at point a and $\tau = 0$ at point o. If B is the half width of foundation, $ac = ad = \alpha_1 B$, $ae = \alpha_2 B$, α_1, α_2 and θ are 3 unknowns to be sought on the basis of minimum principle. Finally a minimum limit load $Q_{\min}$ will be obtained. A computer program to carry out this kind of calculation is very simple. For the same example shown in Fig. 5, the critical values are $\alpha_1 = 0.62$, $\alpha_2 = 0.66$, $\theta = 38°$, and the corresponding mean limit load is $\bar{q}_f = Q_{\min}/B = 106.7$kPa, as shown in Fig. 8. The comparison of these results with those shown in Fig. 5 demonstrates that the wedge equilibrium solution has sufficient accuracy, only an error of 4.2% higher than the exact solution is resulted. In addition the mean shear stress at the interface 8.83kPa is also very close to 9.20kPa obtained from the method of characteristic line, therefore the assumption of $T = 0.5T_f$ is also proved to be adequate.

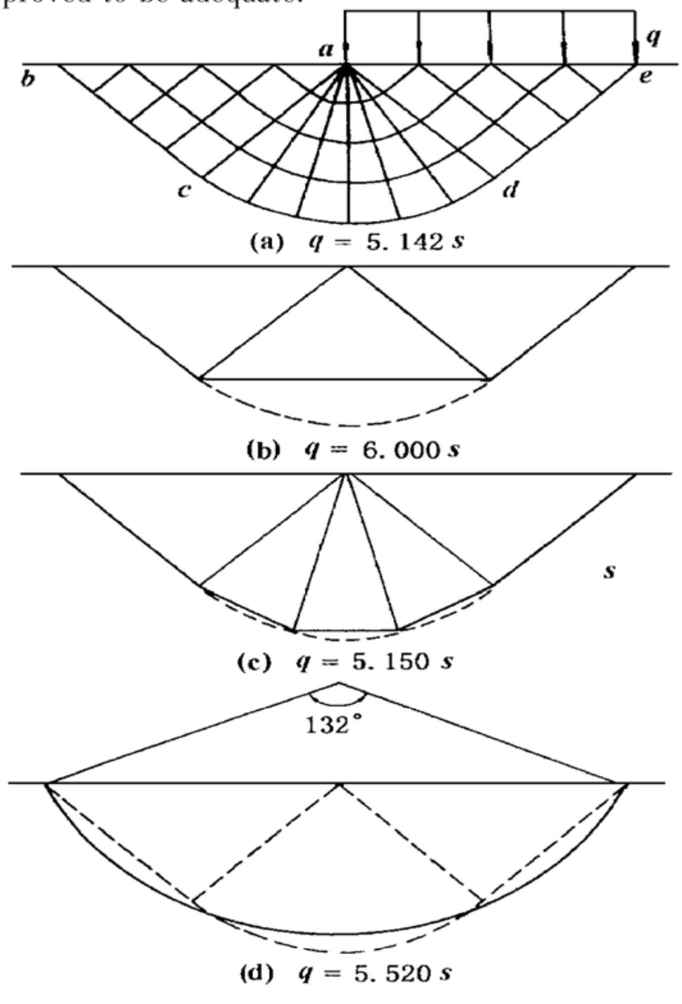


Fig. 6 Limit load on ideal plastic soil

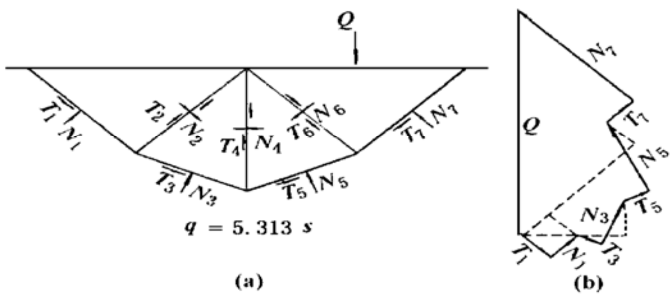


Fig. 7 Force equilibrium between slide wedges

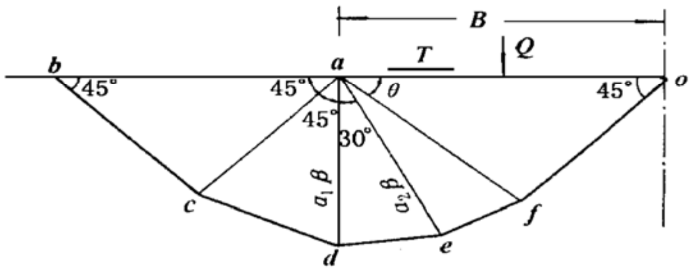


Fig. 8　Slide wedges in reinforced soft ground

As a matter of fact, if the surface load is produced by the weight of embankment, there must be a horizontal force acting in the opposite direction due to lateral earth pressure. Thus, the slide surface should be changed in the case without geosynthetics. Assuming an coefficient of earth pressure of 0.4, the vertical load of 50 kPa will produce a total horizontal force 32.5kN/m and accordingly a mean tangential stress of 6.5 kPa acting to the left will develop when this load is converted to 3m height of embankment with density of soil equal to 1.67t/m^3. For this case a four-wedge solution will give a limit load of q = 86.5kPa with the critical values α_1 = 0.4, α_2 = 0.42, and θ = 28°, as shown in Fig. 9. Comparison of this solution with that shown in Fig. 8 gives an estimation of the effect of reinforcement as an increase of limit load by 23.4%。

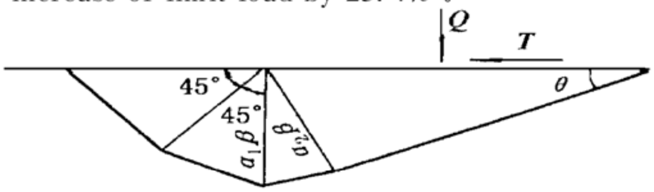


Fig. 9　Slide wedges without reinforcement

6　Factor of safety

In the limit analysis the factor of safety is usually determined as the ratio of limit load to the design load

$$K_1 = \frac{Q}{Q_0} \tag{5a}$$

But in the slope stability analysis the traditional definition is based on the concept of reduction of soil strength

$$K_1 = \frac{s_0}{s} = \frac{c_0 + \sigma_0' \cdot \mathrm{tg}\,\varphi_0}{c + \sigma' \cdot \mathrm{tg}\,\varphi} \tag{5b}$$

where c_0 and φ_0 are design parameters, and c and φ are reduced parameters. According to the effective consolidation stress theory the effective consolidation stress σ' in the limit state is equal to σ_0' in the design state, therefore

$$K_1 = \frac{c_0}{c} = \frac{\mathrm{tg}\,\varphi_0}{\mathrm{tg}\,\varphi} \tag{5c}$$

Meanwhile, it is evident from the force polygon shown in Fig. 7(b) that the length of force vectors T_1, T_2 …… is proportional to the shear strength of soil. Therefore, when shear strength is reduced by K_1 times the limit load also reduced proportionally. That is to say, the definition expressed in Eq. (5a) is exactly equivalent to that of Eq. (5c). But things will get complicated if the embankment body consisting of sand or unsaturated soils will also be involved in the sliding mass, because for these soils the effective consolidation stress concept is not valid, i.e. $\sigma_0' \neq \sigma'$ in Eq. (5b). In addition, if tg φ of embankment soil is reduced by K_1 times, the lateral force T will change and the ratio Q/T changes accordingly, therefore the force polygons will not resemble to each other. But as soon as the reinforced soft ground is concerned the definition expressed in Eq. (5a) is adequate.

The next topic need to be discussed is the factor of safety of geosynthetics against tensile failure. To ensure the integrity of embankment body and to prevent lateral spreading of ground soil, the geosynthetics must have sufficient strength to resist the friction force both from bottom interface and top interface. Therefore the factor of safety can be defined as follows

$$K_2 = \frac{T}{T_1 + T_2} \tag{6}$$

where

$$T_1 = \frac{1}{2}\int_0^B s\,\mathrm{d}l \tag{7}$$

$$T_2 = \frac{1}{2}\gamma K_a H^2 \tag{8}$$

T_1 is the total shear force acting at the bottom and T_2 is that acting on the top, s is the shear strength of soil on the ground surface, K_a is the coefficient of earth pressure and H is height of embankment.

A tentative suggestion is that both K_1 and K_2 must be higher than 1.3. The recommendation is to use at least two layers of geosynthetics, the top layer is aimed to resist the horizontal force from earth pressure of embankment fill and the second one is aimed to bear the shear force induced by the lateral deformation of foundation soil.

7　Application to a test embankment

A test embankment was constructed on the soft ground at Lianyungang harbor to verify the effectiveness of reinforcement[8]. The section without reinforcement failed when the embankment reached its height of 4.04m, while the section reinforced only by one layer of geotextile failed as a result of its rupture when the height of filling is 4.35m. In both cases cracks as wide as 0.3m developed along the center line, and the embankment failed in symmetric pattern.

If the shear strength 22.5kPa of surface soil is taken to assess the tangential force developed at the interface, then T_1 = 22.5 × 11.23/2 = 126kN/m which is much larger than the ultimate tensile strength of geotextile 40kN/m. Even if the minimum strength of soil 8.8kPa is used, T_1 = 49.4kN/m still exceeds its rupture strength. Therefore, the rupture of geotextile is inevitable.

Next we will make an analysis for the failure of the non-reinforced section. Because the embankment was actually broken into two parts, the horizontal force due to earth pressure of embankment can be assumed to be zero. The wedge equilibrium method gives a limit load Q = 735 kN/m, a little larger than the actual load 618kN/m at failure, or the corresponding factor of safety is 1. 19, as shown in Fig. 10, where the computed critical wedge system is also compared with the actual failure surface. These results confirm the appropriateness of the proposed design procedure.

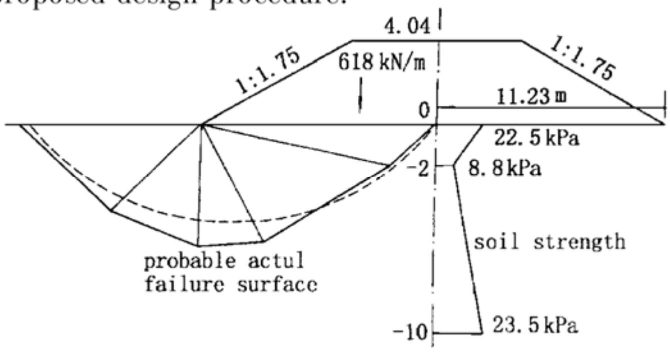


Fig. 10 Failure of a test embankment

8 Conclusions

(1) For the soft ground reinforced by geosynthetics with sufficient tensile strength the only possible mode of failure is lateral spreading of soil accompanied with subsidence of the remained embankment, and the rotation slide accepted in the conventional circular arc method is practically impossible.

(2) The installation of geosynthetics can greatly increase the bearing capacity of soft ground because of the change of shear stress direction along the ground surface.

(3) The proposed force equilibrium method is a development of slide wedge method traditionally used in the soil mechanics. It is simple, but has sufficient accuracy.

References

1 石湘森. 土工织物在大目涂堤基中的应用. 地基处理, 1992, **3**(1): 42~ 50.

2 刘吉福, 龚晓南, 王盛源. 一种考虑土工织物抗滑作用的稳定分析方法. 地基处理, 1996, **7**(2): 1~ 5.

3 殷建华. A one-dimensional geosynthetic-reinforced foundation model with non-linear spring support. Chinese Journal of Geotechnical Engineering, 1998, **20**(1): 76~ 79.

4 沈珠江. 散粒体极限平衡理论及其应用. 水利学报, 1962 (3): 22~ 36.

5 沈珠江. 饱和粘性土的极限平衡理论. 见: 第一届土力学及基础工程学术会议论文选集. 天津: 中国工业出版社, 1962. 193~ 201.

6 朱百里, 沈珠江. 计算土力学. 第五章. 上海科技出版社, 1990.

7 陈祖煜. 边坡稳定分析的塑性力学上限解. 见: 第七届土力学及基础工程学术会议论文集. 北京: 中国建筑工业出版社, 1994 . 484~ 488.

8 赵九斋, 龙国英, 徐啸海等. 土工织物加固路基和天然路基对称破坏及其分析. 岩土工程学报, 1991, **13**(2): 73~ 81.